Probing quantum vacuum geometrical effects with cold atoms

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Collaborators

- Theory on lateral Casimir-Polder forces:

  Astrid Lambrecht (LKB, Paris)

  Paulo Maia Neto (Rio de Janeiro)

  Serge Reynaud (LKB, Paris)

- Experiments on atom-surface interactions:

  Malcolm Boshier (Los Alamos)

  Matt Blain (Sandia National Laboratories)

Outline of this talk

- Brief review of theory and experiments on van der Waals/Casimir-Polder forces
- Casimir-Polder forces within scattering theory
- Lateral Casimir-Polder forces beyond PFA
- Cold atoms for probing CP forces beyond PFA

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- Brief review of theory and experiments on van der Waals/Casimir-Polder forces
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- Cold atoms for probing CP forces beyond PFA
- Metamaterials for engineering Casimir forces

Casimir-Polder forces

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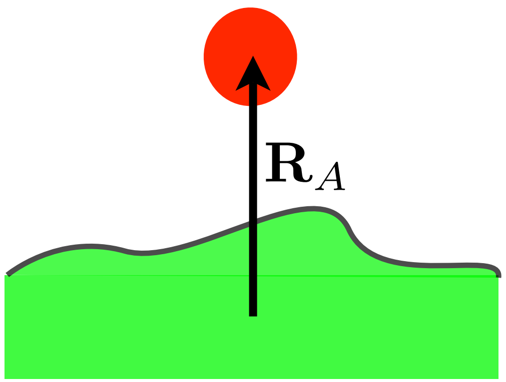
vdW - CP interaction

Casimir and Polder (1948)

The interaction energy between a ground-state atom and a surface is given by

$$\boxed{U_{\rm CP}(\mathbf{R}_A) = \frac{\hbar}{c^2\epsilon_0}\int_0^\infty \frac{d\xi}{2\pi}\xi^2\alpha(i\xi)\,\mathrm{Tr}\,\mathbf{G}(\mathbf{R}_A,\mathbf{R}_A,i\xi)}$$



Atomic polarizability: $\alpha(\omega) = \lim_{\epsilon\to 0}\frac{2}{3\hbar}\sum_k \frac{\omega_{k0}|\mathbf{d}_{0k}|^2}{\omega_{k0}^2-\omega^2-i\omega\epsilon}$

Scattering Green tensor: $\left(\nabla\times\nabla\times-\frac{\omega^2}{c^2}\epsilon(\mathbf{r},\omega)\right)\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}')$

Casimir-Polder forces

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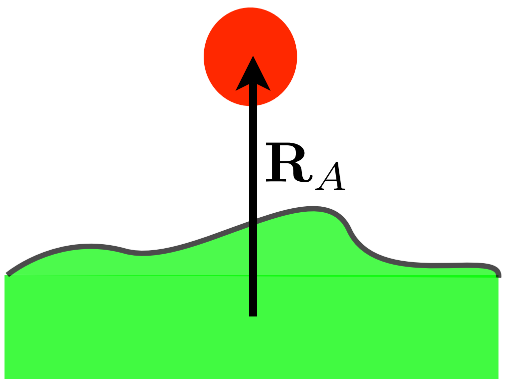

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Eg: Atom near planar surface @ T=0

Non-retarded (vdW) limit $z_A \ll \lambda_A$

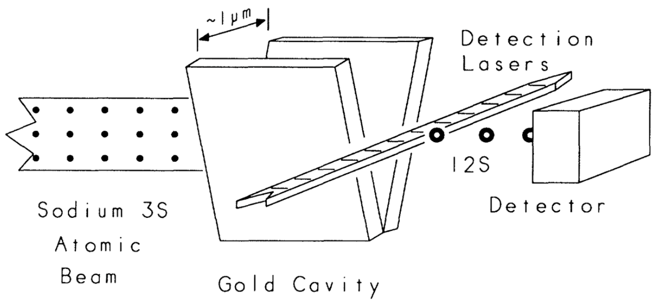
$$U_{\mathrm{vdW}}(z_A) = -\frac{\hbar}{8\pi\epsilon_0}\frac{1}{z_A^3}\int_0^\infty \frac{d\xi}{2\pi}\alpha(i\xi)\frac{\epsilon(i\xi)-1}{\epsilon(i\xi)+1}$$

Retarded (CP) limit $z_A \gg \lambda_A$

$$U_{\mathrm{CP}}(z_A) = -\frac{3\hbar c\alpha(0)}{8\pi}\frac{1}{z_A^4}\frac{\epsilon_0 - 1}{\epsilon_0 + 1}\phi(\epsilon_0)$$

Modern experiments

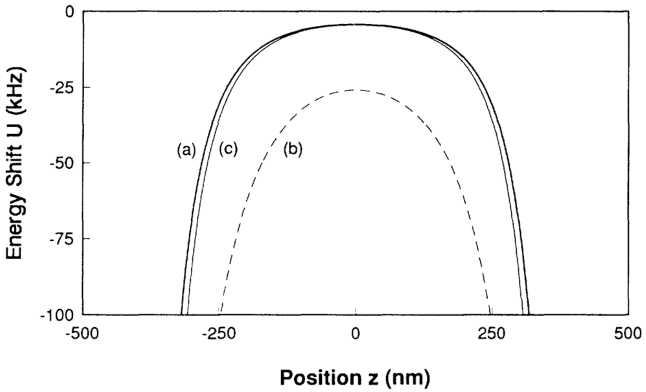
- ## Deflection of atoms



L= 0.7-1.2 um

Exp-Th agreement @ 10%

Hinds et al (1993)



$$U_{CP} = -\frac{1}{4\pi\epsilon_0}\frac{\pi^3\hbar c\alpha(0)}{L^4}\left[\frac{3-2\cos^2(\pi z/L)}{8\cos^4(\pi z/L)}\right]$$

Modern experiments

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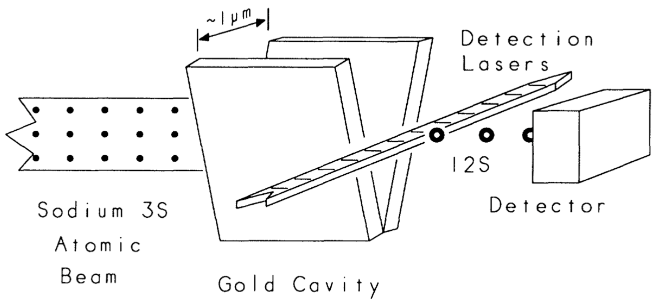

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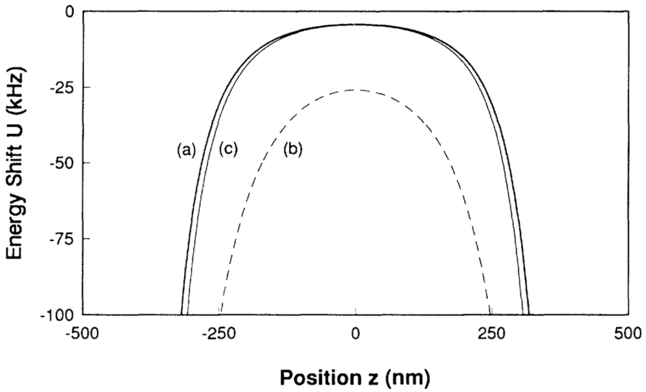

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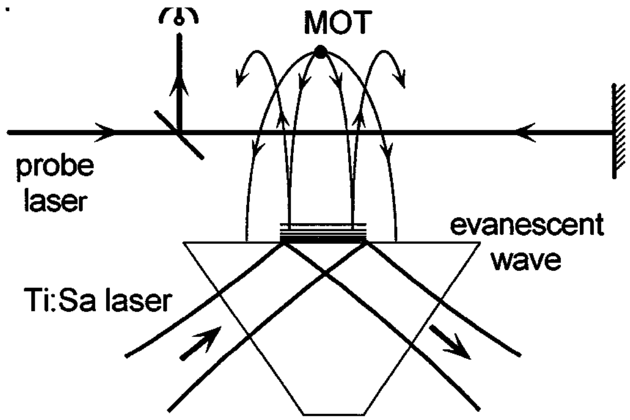
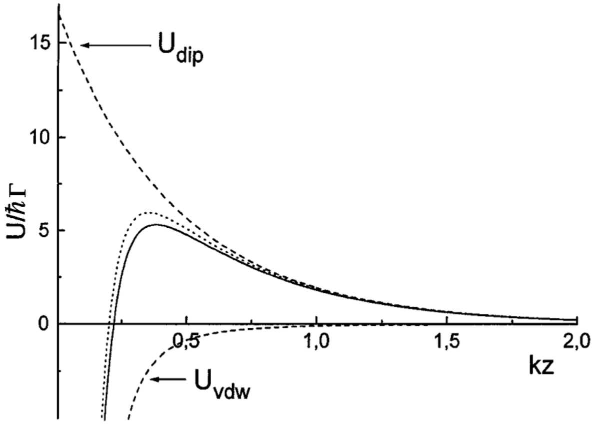
Classical reflection on atomic mirror

Aspect et al (1996)



$$U_{\rm dip} = \frac{\hbar}{4}\frac{\Omega^2}{\Delta}e^{-2kz}$$

$$U_{\rm vdW} = -\frac{\epsilon-1}{\epsilon+1}\frac{1}{48\pi\epsilon_0}\frac{D^2}{z^3}$$



Exp-Th agreement @ 30%

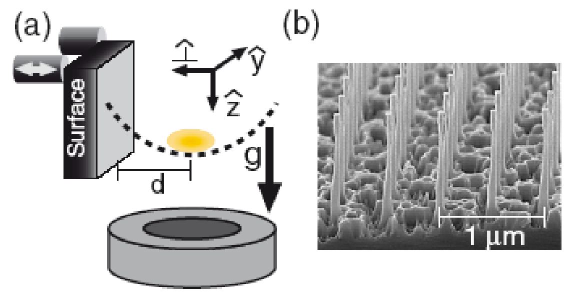
Modern experiments (cont'd)

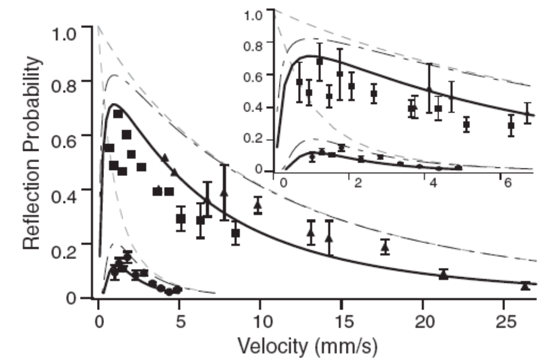
Quantum reflection

Wave-nature of atoms implies that slow atoms can reflect from purely attractive potentials

$$k = \sqrt{k_0^2 - 2mU/\hbar^2} \qquad \phi = \frac{1}{k^2}\frac{dk}{dr} > 1$$

$$U = -C_n/r^n \ (n > 2)$$





Shimizu (2001)

Ketterle et al (2006)

DeKievet et al (2003)

Modern experiments (cont'd)

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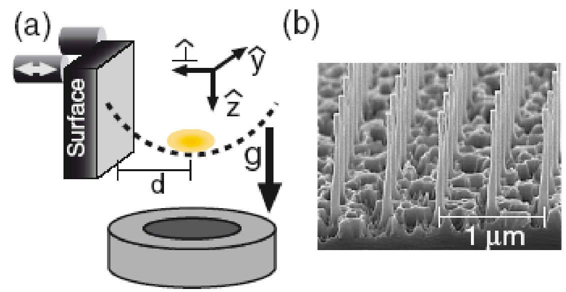

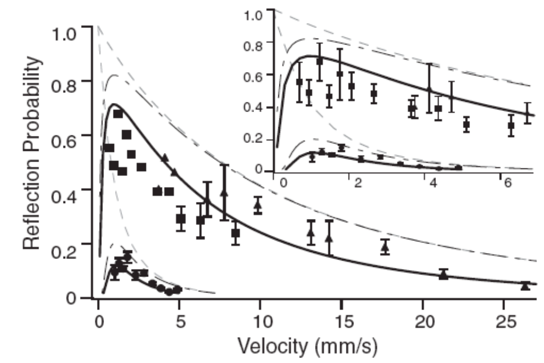

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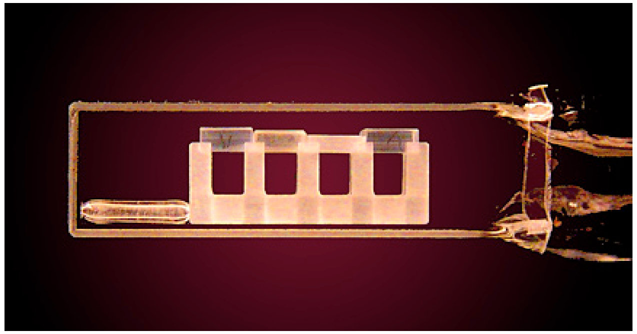
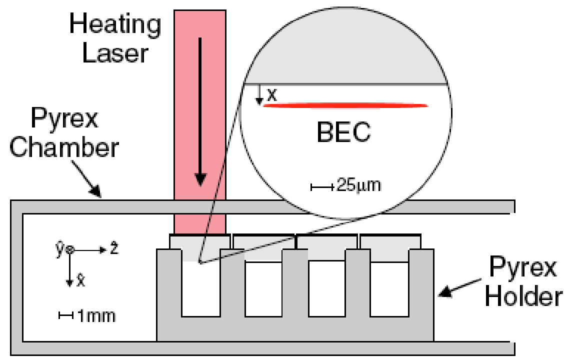
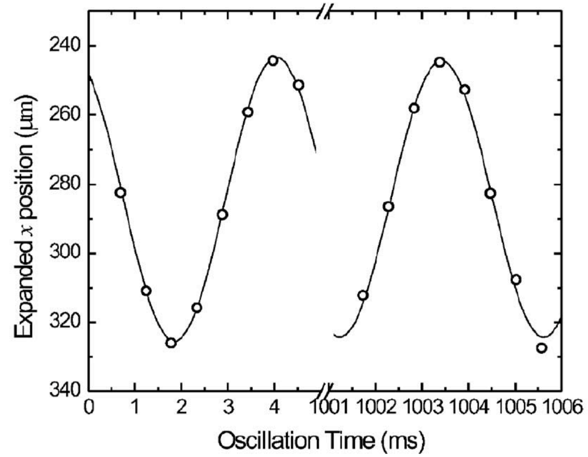
Shimizu (2001) Ketterle et al (2006)
DeKievet et al (2003)

BEC oscillator

Cornell et al (2007)

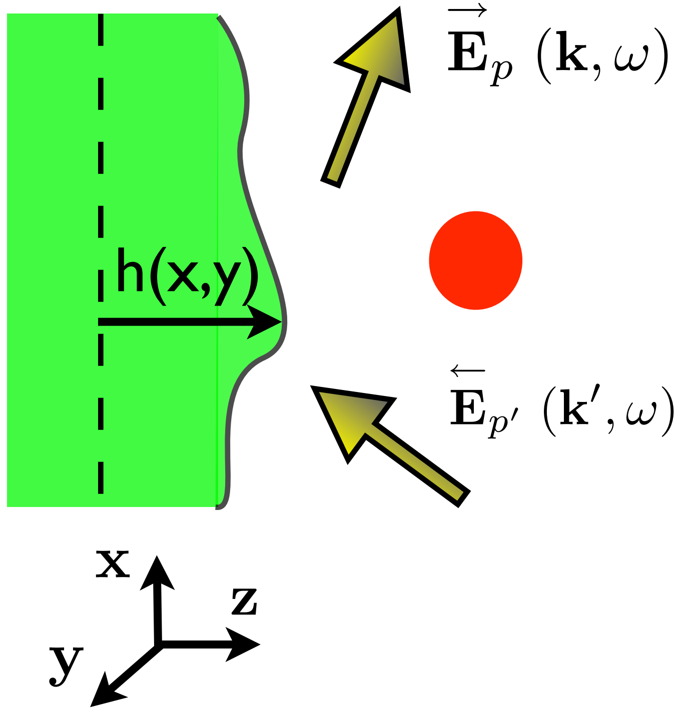






$$\gamma_x \equiv \frac{\omega_x - \omega_x'}{\omega_x} \simeq -\frac{1}{2\omega_x^2 m}\partial_x^2 U^*$$

CP within scattering theory



Output fields: $\overrightarrow{\mathbf{E}}\,(\mathbf{R},\omega) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{-i\mathbf{k}\cdot\mathbf{r}}\,\overrightarrow{\mathbf{E}}\,(\mathbf{k},z,\omega)$

$$\overrightarrow{\mathbf{E}}\,(\mathbf{k},z,\omega) = [\overrightarrow{E}_{\mathrm{TE}}\,(\mathbf{k},\omega)\,\hat{\epsilon}^{+}_{\mathrm{TE}}(\mathbf{k}) + \overrightarrow{E}_{\mathrm{TM}}\,(\mathbf{k},\omega)\,\hat{\epsilon}^{+}_{\mathrm{TM}}(\mathbf{k})]\,e^{ik_z z}$$

$$\hat{\epsilon}^{+}_{\mathrm{TE}}(\mathbf{k}) = \mathbf{z}\times\mathbf{k} \qquad \hat{\epsilon}^{+}_{\mathrm{TM}}(\mathbf{k}) = \hat{\epsilon}^{+}_{\mathrm{TE}}(\mathbf{k})\times\mathbf{K} \qquad (\mathbf{K} = \mathbf{k} + k_z\mathbf{z})$$

Input fields: idem with $k_z \rightarrow -k_z$

Input and output fields related via reflection operators

$$\boxed{\overrightarrow{\mathbf{E}}_p\,(\mathbf{k},\omega) = \int \frac{d^2\mathbf{k}'}{(2\pi)^2}\sum_{p'}\langle\mathbf{k},p|\mathcal{R}(\omega)|\mathbf{k}',p'\rangle\;\overleftarrow{\mathbf{E}}_{p'}\,(\mathbf{k}',\omega)}$$

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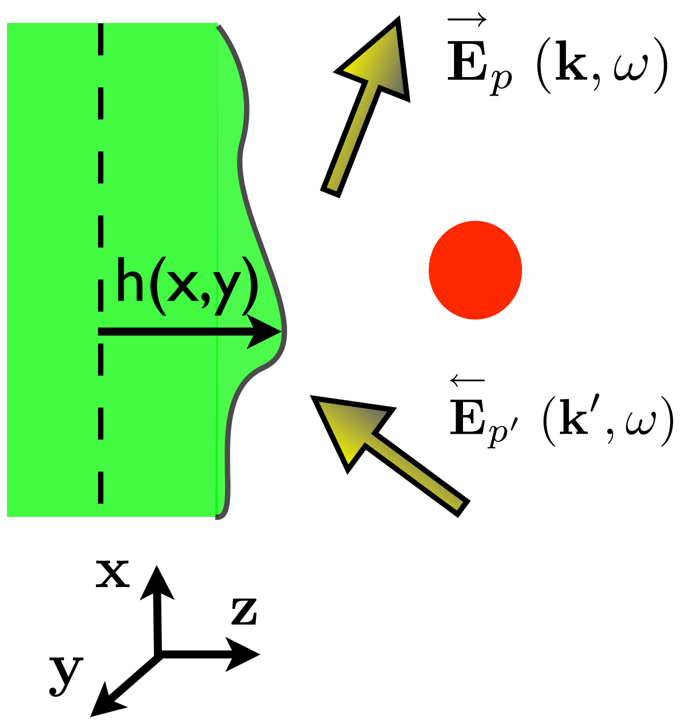

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Casimir-Polder force:

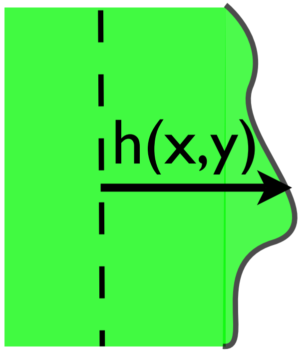
$$\boxed{U_{\mathrm{CP}}(\mathbf{R}_A) = \frac{\hbar}{c^2\epsilon_0}\int_0^\infty \frac{d\xi}{2\pi}\xi^2\alpha(i\xi)\int\frac{d^2\mathbf{k}}{(2\pi)^2}\int\frac{d^2\mathbf{k}'}{(2\pi)^2} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_A} e^{-(\kappa+\kappa')z_A}\frac{1}{2\kappa'}\sum_{p,p'}\hat{\epsilon}^{+}_{p}(\mathbf{k})\cdot\hat{\epsilon}^{-}_{p'}(\mathbf{k}')\,R_{p,p'}(\mathbf{k},\mathbf{k}')}$$

with $\kappa \equiv \sqrt{\xi^2/c^2 + k^2}$ and $R_{p,p'}(\mathbf{k},\mathbf{k}')$ dependent on material properties at freq. $i\xi$

Specular/non specular scattering

In order to treat a general rough or corrugated surface, we make a perturbative expansion in powers of h(x,y)

$$\mathcal{R} = \mathcal{R}^{(0)} + \mathcal{R}^{(1)} + \dots$$



- Specular reflection:

$$\boxed{\langle \mathbf{k}, p | \mathcal{R}^{(0)} | \mathbf{k}', p' \rangle = (2\pi)^2 \delta^{(2)}(\mathbf{k} - \mathbf{k}') \, \delta_{p,p'} \, r_p(\mathbf{k}, \xi)}$$

Fresnel coefficients $\quad r_{\mathrm{TE}} = \frac{\kappa - \kappa_t}{\kappa + \kappa_t} \qquad r_{\mathrm{TM}} = \frac{\epsilon(i\xi)\kappa - \kappa_t}{\epsilon(i\xi)\kappa + \kappa_t} \qquad (\kappa_t = \sqrt{\epsilon(i\xi)\xi^2/c^2 + k^2})$

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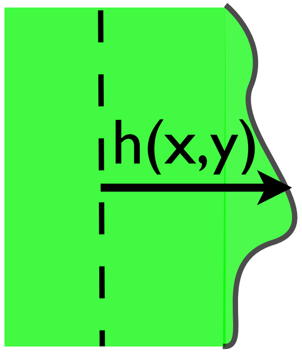

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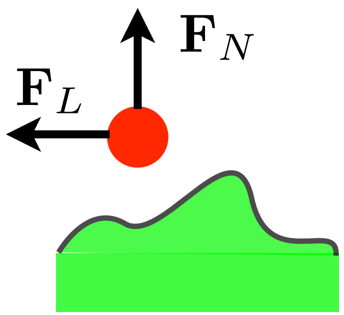
Non-specular reflection:

$$\boxed{\langle \mathbf{k}, p | \mathcal{R}^{(1)} | \mathbf{k}', p' \rangle = R_{p,p'}(\mathbf{k}, \mathbf{k}')\, H(\mathbf{k} - \mathbf{k}')}$$ ⟵ Fourier transform of h(x,y)

The non-specular reflection matrices depend on the geometry and material properties.

They can be obtained from the Extinction Theorem of electromagnetism using the Rayleigh approximation. This says that all incoming fields are reflected back to infinity, which requires small slopes of the profile h(x,y) Greffet (1988), Reynaud et al (2005)

Lateral Casimir-Polder force



$$\boxed{U_{\rm CP} = U_{\rm CP}^{(0)}(z_A) + U_{\rm CP}^{(1)}(z_A, x_A)}$$

- **Normal CP force:** $$U_{\rm CP}^{(0)}(z_A) = \frac{\hbar}{c^2\epsilon_0}\int_0^\infty \frac{d\xi}{2\pi}\xi^2\alpha(i\xi)\int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{1}{2\kappa}\sum_p \hat{\epsilon}_p^+\cdot\hat{\epsilon}_p^-\ r_p(\mathbf{k},\xi)\ e^{-2\kappa z_A}$$

- **Lateral CP force:** $$U_{\rm CP}^{(1)}(z_A, x_A) = \int\frac{d^2\mathbf{k}}{(2\pi)^2}\ e^{i\mathbf{k}\cdot\mathbf{r}_A}\ g(\mathbf{k}, z_A)\ H(\mathbf{k})$$

Response function g:

$$g(\mathbf{k}, z_A) = \frac{\hbar}{c^2\epsilon_0}\int_0^\infty \frac{d\xi}{2\pi}\xi^2\alpha(i\xi)\int\frac{d^2\mathbf{k}'}{(2\pi)^2}\ a_{\mathbf{k}',\mathbf{k}'-\mathbf{k}}(z_A,\xi)$$

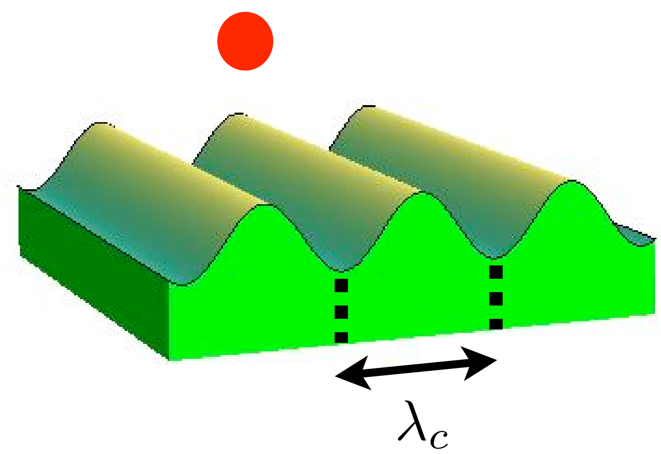
$$a_{\mathbf{k}',\mathbf{k}''} = \sum_{p',p''}\hat{\epsilon}_{p'}^+\cdot\hat{\epsilon}_{p''}^-\ \frac{e^{-(\kappa'+\kappa")z_A}}{2\kappa''}\ R_{p',p''}(\mathbf{k}',\mathbf{k}'')$$

Our approach is perturbative in h(x,y), which should be the smallest length scale in the problem $h \ll z_A, \lambda_c, \lambda_A, \lambda_0$

Sinusoidal corrugation

Uni-axial corrugation: $h(x,y) = h_0 \cos(k_c\, x)$

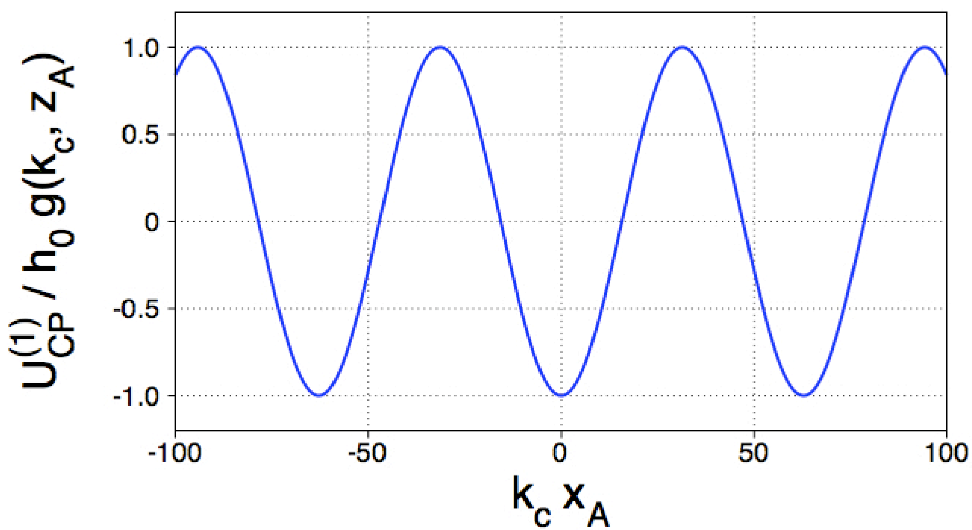
Corrugation period: $\lambda_c = 2\pi/k_c$



Lateral Casimir-Polder force:

$$U^{(1)}_{\rm CP} = h_0 \cos(k_c\, x_A)\; g(k_c, z_A)$$

$$\mathbf{F}_L = k_c h_0 \sin(k_c\; x_A)\; g(k_c, z_A)\; \mathbf{x}$$



We will show below that $g(k_c, z_A) < 0$, so that the lateral force brings the atom to the neighborhood of one of the crests

Proximity force approximation

- The PFA corresponds to approximating the CP energy by its expression for the planar case with a "local" distance $z_A - h(\mathbf{r}_A)$

$$U_{\rm CP}(\mathbf{R}_A) \approx U_{\rm CP}^{(0)}(z_A - h(\mathbf{r}_A)) \approx U_{\rm CP}^{(0)}(z_A) - h(\mathbf{r}_A)\, U_{\rm CP}^{(0)'}(z_A)$$

- The pairwise summation approach is also approximate, since Casimir forces are not additive, expect in the special case of very dilute media.

- The PFA corresponds to the limiting case where the corrugation is very smooth with respect to the other length scales:

$$\boxed{k_c\, z_A \ll 1 \qquad \text{[PFA]}}$$

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$$\boxed{k_c\, z_A \ll 1 \quad \text{[PFA]}}$$

- Given that the lateral CP potential is $U_{\rm CP}^{(1)} = h(\mathbf{r}_A)\; g(k_c, z_A)$, we obtain

"proximity force theorem"

$$g(k_c \to 0, z_A) = -\frac{dU_{\rm CP}^{(0)}(z_A)}{dz_A}$$

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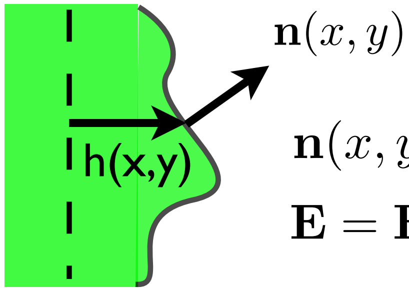
$$g(k_c \to 0, z_A) = -\frac{dU^{(0)}_{\rm CP}(z_A)}{dz_A}$$

- Deviations from PFA can be measured by the ratio

$$\boxed{\rho \equiv \frac{g(k_c, z_A)}{g(0, z_A)}}$$

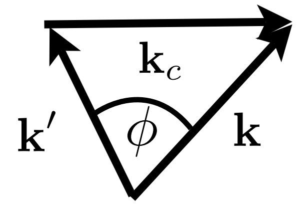
Perfect reflectors



$$\mathbf{n}(x,y) \times \mathbf{E}(x,y,h(x,y)) = 0$$

$$\mathbf{E} = \mathbf{E}^{(0)} + \mathbf{E}^{(1)} + \dots$$

$$\mathbf{R}^{(1)}(\mathbf{k},\mathbf{k}';\xi) = -2 \begin{pmatrix} \kappa' C & \xi S/c \\ \frac{\xi\kappa'}{c\kappa} S & -\frac{kk'}{\kappa} - \frac{\xi^2}{c^2\kappa} C \end{pmatrix}$$

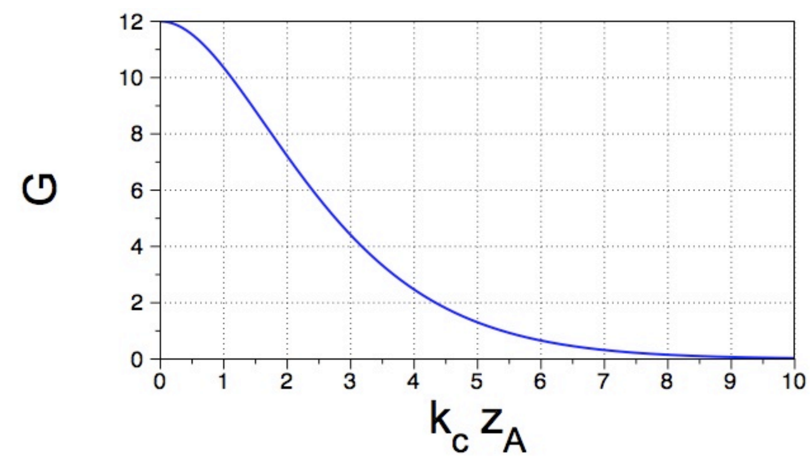


$$C = \cos\phi$$
$$S = \sin\phi$$

- vdW response function $(z_A \ll \lambda_A)$

$$g(k_c, z_A) = -\frac{\hbar G(k_c z_A)}{64\pi^2\epsilon_0 z_A^4} \int_0^\infty d\xi \alpha(i\xi)$$

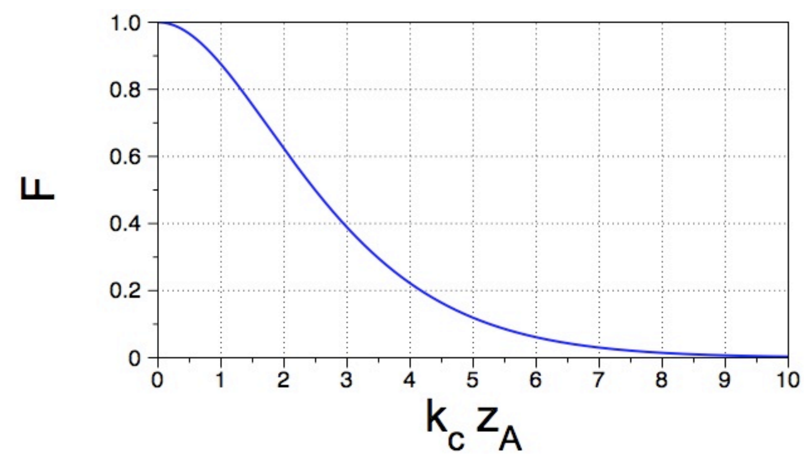
$$G(\mathcal{Z}) = \mathcal{Z}^2[2K_2(\mathcal{Z}) + \mathcal{Z}K_3(\mathcal{Z})]$$



- CP response function $(z_A \gg \lambda_A)$

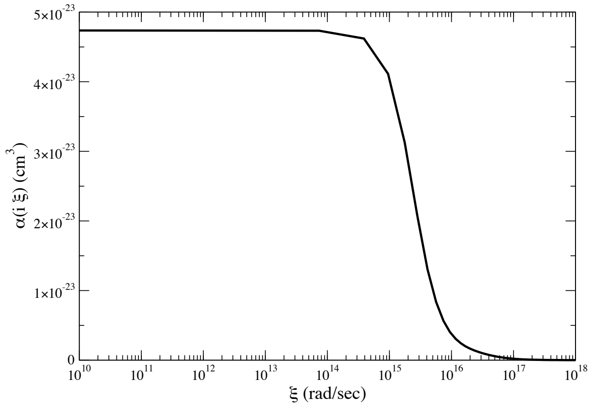
$$g(k_c, z_A) = -\frac{3\hbar c\alpha(0)}{8\pi^2 z_A^5} F(k_c z_A)$$

$$F(\mathcal{Z}) = e^{-\mathcal{Z}}(1 + \mathcal{Z} + 16\mathcal{Z}^2/45 + \mathcal{Z}^3/45)$$

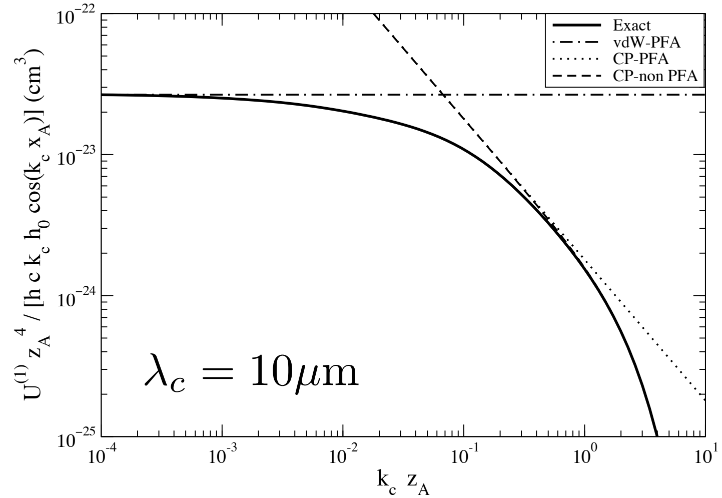


Perfect reflectors (cont'd)

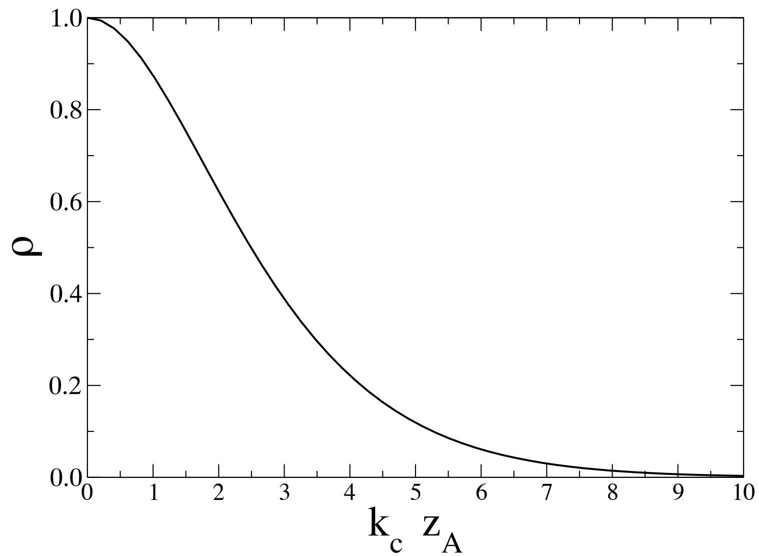
- Dynamic polarizability of Rb
  Babb et al (1999)



- Lateral potential energy $U^{(1)}$
  Rb + sine corrug. + perf. reflector



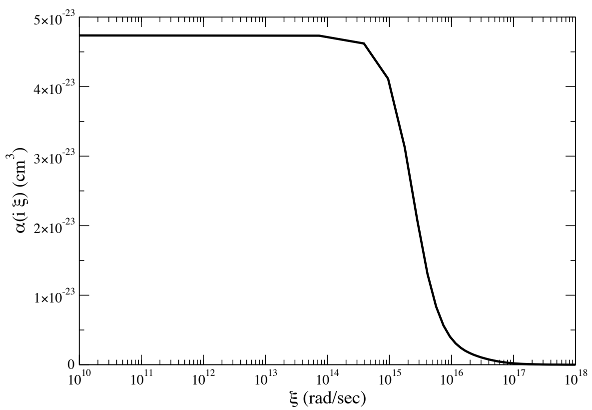
- Deviations from PFA



Perfect reflectors (cont'd)

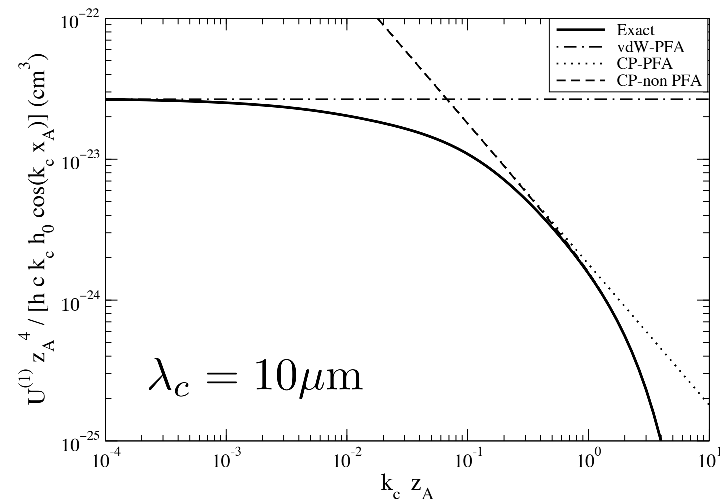
- Dynamic polarizability of Rb

  Babb et al (1999)

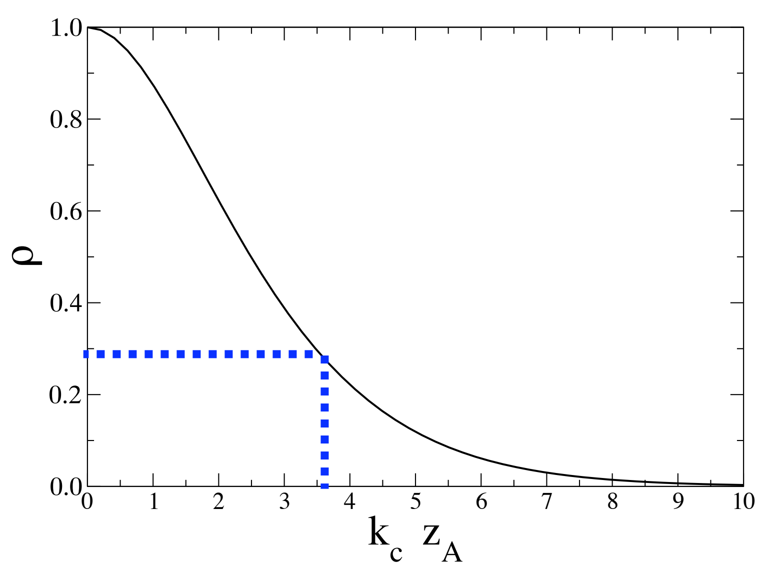


- Lateral potential energy $U^{(1)}$

  Rb + sine corrug. + perf. reflector



- Deviations from PFA



Example:

atom-surface distance $z_A = 2\mu\mathrm{m} \gg \lambda_A$

corrugation wavelength $\lambda_c = 3.5\mu\mathrm{m}$

$\Rightarrow$ $\rho \approx 30\%$

PFA largely overestimates the magnitude of the lateral effect !

Real materials

- Calculation of $R^{(1)}_{p,p'}(\mathbf{k},\mathbf{k}',\xi)$ in terms of $\epsilon(i\xi)$ of bulk materials   Reynaud et al (2005)

$$R^{(1)}_{\mathrm{TE,TE}}(\mathbf{k},\mathbf{k}';\xi) = 2\kappa\, C\, h_{\mathrm{TE,TE}}(\mathbf{k},\mathbf{k}',\xi)$$

$$R^{(1)}_{\mathrm{TE,TM}}(\mathbf{k},\mathbf{k}';\xi) = 2\kappa S\,\frac{c\kappa'_t}{\sqrt{\epsilon}\xi}\, h_{\mathrm{TE,TM}}(\mathbf{k},\mathbf{k}',\xi)$$

$$R^{(1)}_{\mathrm{TM,TM}}(\mathbf{k},\mathbf{k}';\xi) = -2\kappa\frac{\epsilon k k' + \kappa_t\kappa'_t C}{(\frac{\xi}{c})^2 - (\epsilon+1)\kappa^2}\, h_{\mathrm{TM,TM}}(\mathbf{k},\mathbf{k}',\xi)$$

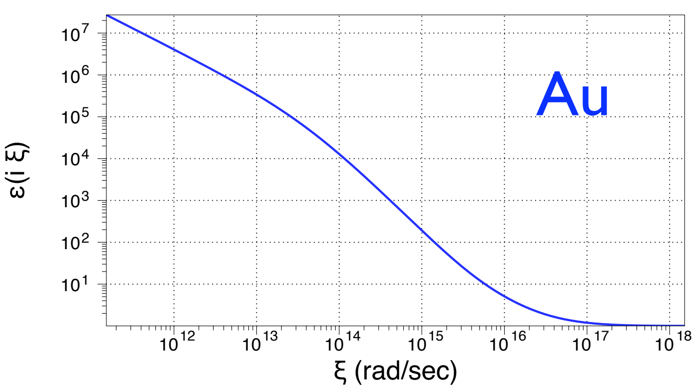
$$R^{(1)}_{\mathrm{TM,TE}}(\mathbf{k},\mathbf{k}';\xi) = \frac{2\sqrt{\epsilon}\kappa\kappa_t\frac{\xi}{c}S}{(\frac{\xi}{c})^2 - (\epsilon+1)\kappa^2} h_{\mathrm{TM,TE}}(\mathbf{k},\mathbf{k}',\xi)$$

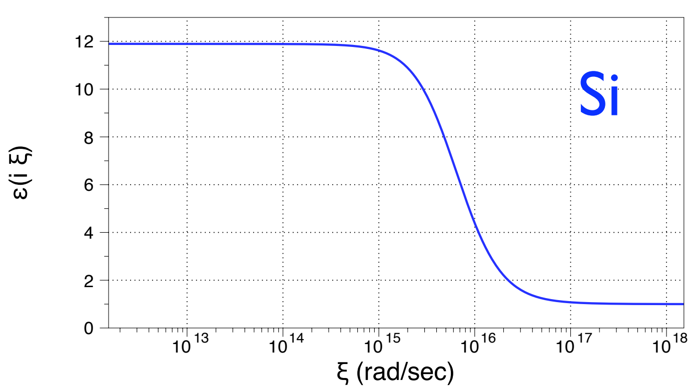
$$h_{pp'}(\mathbf{k},\mathbf{k}') = \frac{r^p(\mathbf{k})t^{p'}(\mathbf{k}')}{t^p(\mathbf{k})}$$

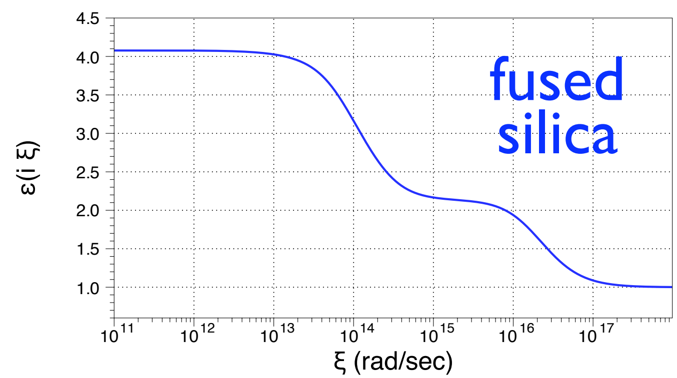
$r^p(\mathbf{k},\xi)$ & $t^p(\mathbf{k},\xi)$

reflection and transmission Fresnel coefficients for a plane inter-phase

- Optical data + Kramers-Kronig relations





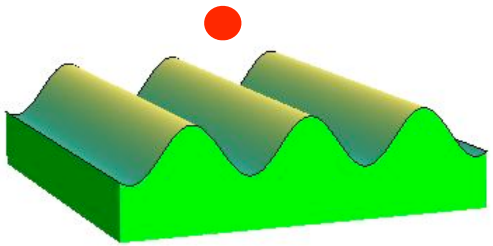


- Deviations from PFA

Just as for perfect reflectors, PFA largely overestimates the lateral CP force. (more later)

Atoms as local probes

In contrast to the case of the lateral Casimir force between corrugated surfaces, an atom is a local probe of the lateral Casimir-Polder force. Deviations from the PFA can be much larger than for the force between two surfaces!



- Before we described large deviations from PFA for a sinusoidal corrugated surface.

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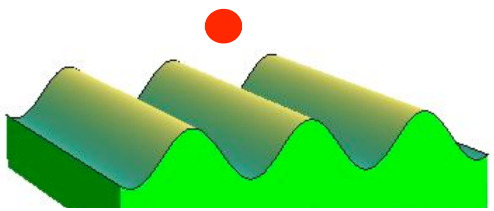
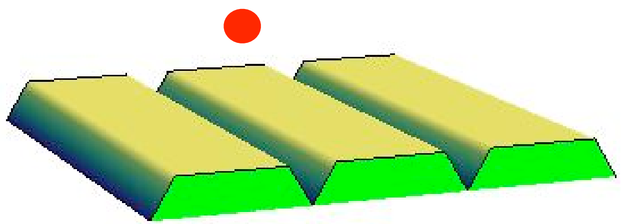


- [x] Even larger deviations from PFA can be obtained for a periodically grooved surface.

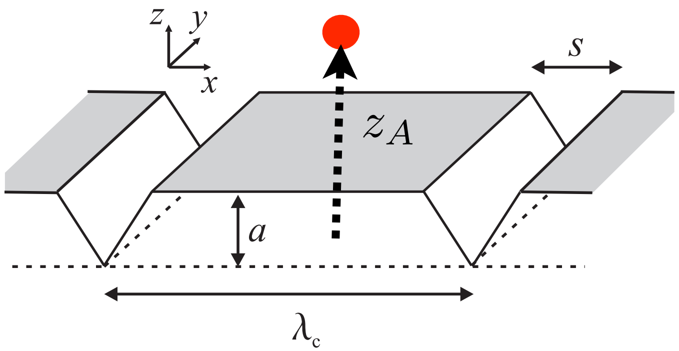


- If the atom is located above one plateau, the PFA predicts that the lateral Casimir-Polder force should vanish, since the energy is thus unchanged in a small lateral displacement.
- A non-vanishing force appearing when the atom is moved above the plateau thus clearly signals a deviation from PFA!

CP energy for grooved surface

- Surface profile for periodical grooved corrugation

$$h(x) = a\left(1 - \frac{s}{2\lambda_c}\right) + \frac{2a\lambda_c}{\pi^2 s}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1-\cos(n\pi s/\lambda_c)}{n^2}\cos\left(\frac{2\pi n x}{\lambda_c}\right)$$



- Single-atom lateral CP energy: it can be easily calculated using that the first order lateral CP energy $U^{(1)}_{\rm CP}(\mathbf{R}_A) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot\mathbf{r}_A}\, g(\mathbf{k}, z_A)\, H(\mathbf{k})$ is linear in $H(\mathbf{k})$

$$\boxed{U^{(1)}_{\rm CP}(x_A, z_A) = a\left(1 - \frac{s}{2\lambda_c}\right) g(0, z_A) + \frac{2a\lambda_c}{\pi^2 s}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1-\cos(nk_c s/2)}{n^2}\, g(nk_c, z_A)\, \cos(nk_c x_A)}$$

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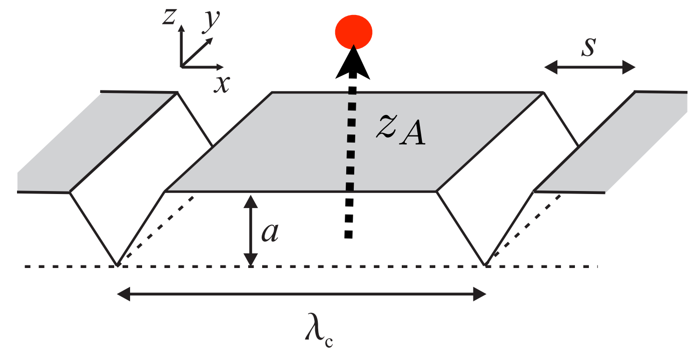

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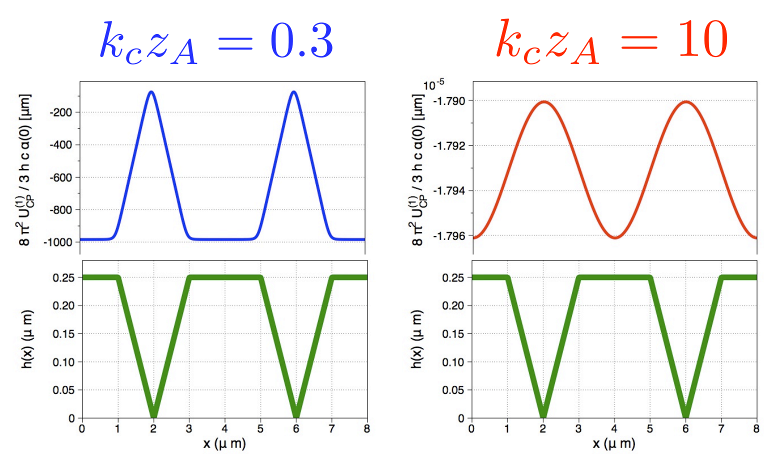
The PFA is recovered when the response function $g(nk_c, z_A)$ may be replaced by $g(0, z_A)$ for all values of n significantly contributing to the profile $h(x)$

When $k_c z_A \gg 1$, the exponential decrease for g implies that the $n = 1$ term dominates the sum, and the potential is approximately sinusoidal, with an effective amplitude

$$h_0 = \frac{2a\lambda_c}{\pi^2 s}\left(1 - \cos(k_c s/2)\right)$$



Eg: for $s = \lambda_c/2$, this gives $h_0 = 100\text{nm} \leftrightarrow a = 250\text{nm}$

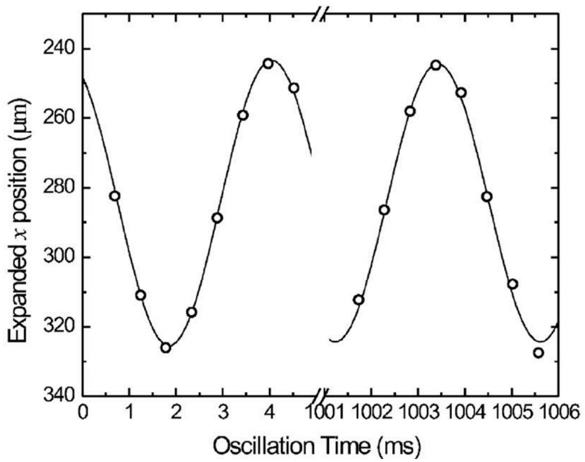
BEC as a field sensor

Novel cold atoms and nano-fabrication techniques offer exciting experimental possibilities to probe quantum vacuum effects. We consider two possible experimental set-ups:

BEC oscillator

- The normal component of Casimir-Polder force $U^{(0)}_{\mathrm{CP}}(z)$ shifts the normal dipolar oscillation frequency of a BEC trapped above a surface

Antezza et al (2004)   Cornell et al (2005, 2007)



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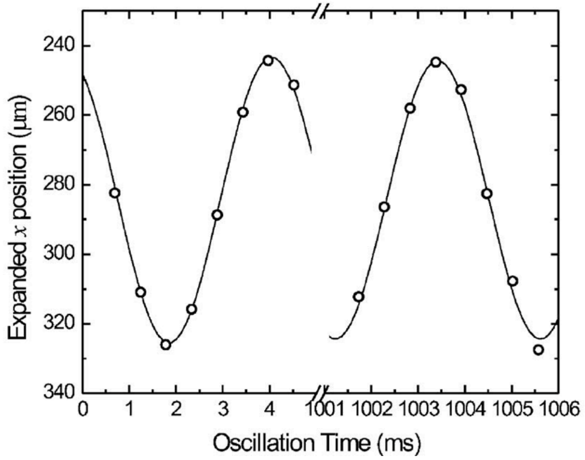
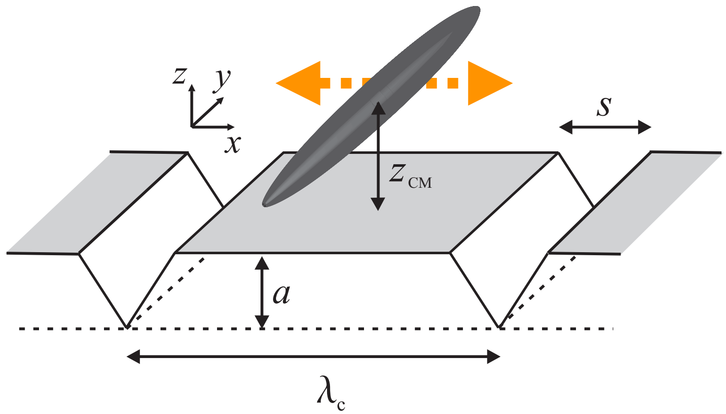


- In order to measure the lateral component $U^{(1)}_{\rm CP}(x,z)$, a cigar-shaped BEC could be trapped parallel to the corrugation lines, and the lateral dipolar oscillation measured as a function of time



$$V(\mathbf{r}) = V_{\rm ho}(\mathbf{r}) + U_{\rm CP}(\mathbf{r})$$

$$V_{\rm ho}(\mathbf{r}) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \quad \omega_y \ll \omega_x = \omega_z$$

Lateral frequency shift:

$$\omega^2_{x,\rm CM} = \omega_x^2 + \frac{1}{m}\int dx dz\, n_0(x,z)\frac{\partial^2}{\partial x^2} U^{(1)}_{\rm CP}(x,z)$$

BEC as a field sensor (cont'd)

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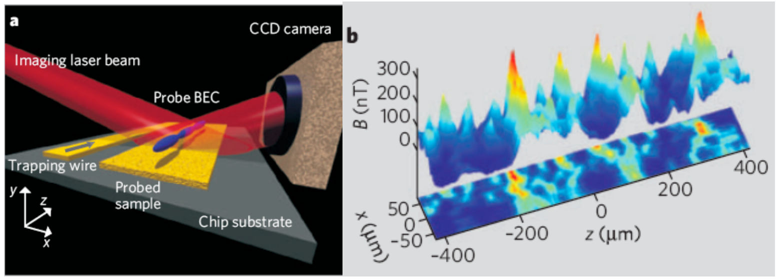
Density variations of a BEC above an atom chip

- For a quasi one-dimensional BEC, the potential is related to the 1D density profile as

$$\boxed{V_{\mathrm{ho}}(x) + U_{\mathrm{CP}}(x) = -\hbar\omega_x\sqrt{1 + 4a_{\mathrm{scat}}n_{1d}(x)}}$$



Measurement of the magnetic field variations along a current-carrying wire

Schmiedmayer et al (2005)

Single shot sensitivity for potential measurement:

$$\Delta U = \frac{\gamma \Delta N}{\rho_0^2 x_0} \;;\; \gamma \equiv \frac{2\hbar^2}{m} a_{\mathrm{scat}}$$

$$\Delta U \simeq 10^{-14}\,\mathrm{eV}\ (@\,\omega_x/2\pi = 300\,\mathrm{Hz})$$

$\Delta N \simeq 4$ atoms per pixel (detection imaging noise)

x_0 longitudinal spatial resolution

ρ_0 transverse spatial resolution

BEC as a field sensor (cont'd)

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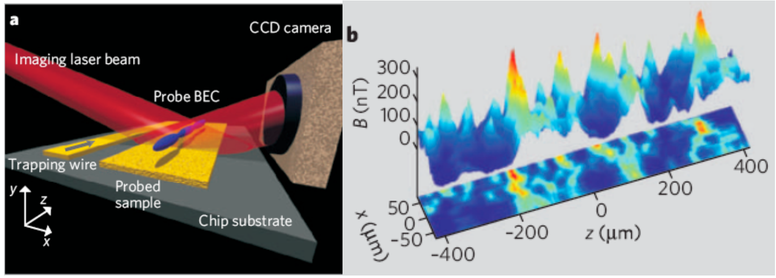

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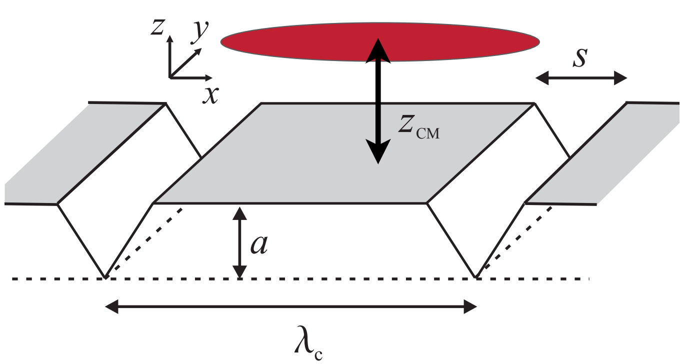
x_0 longitudinal spatial resolution

ρ_0 transverse spatial resolution

For the lateral CP force, perfect conductor, sinusoidal corrugation ($a = 100{\rm nm}$), distance $z_A = 2\mu{\rm m}$, PFA limit $(k_c z_A \ll 1)$

$$\Delta U^{(1)}_{\rm CP} \simeq 10^{-14}\,{\rm eV}$$

- To measure the lateral CP force, the elongated BEC should be aligned along the x-direction, and a density modulation along this direction above the plateau would be a signature of a nontrivial (non-PFA) geometry effect.



Frequency shift for single atom

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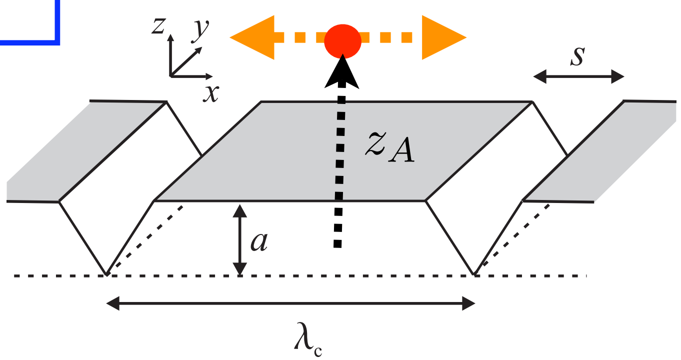
The relative frequency shift is defined as $\boxed{\gamma_0 \equiv \dfrac{\omega_{x,\mathrm{CM}} - \omega_x}{\omega_x}}$

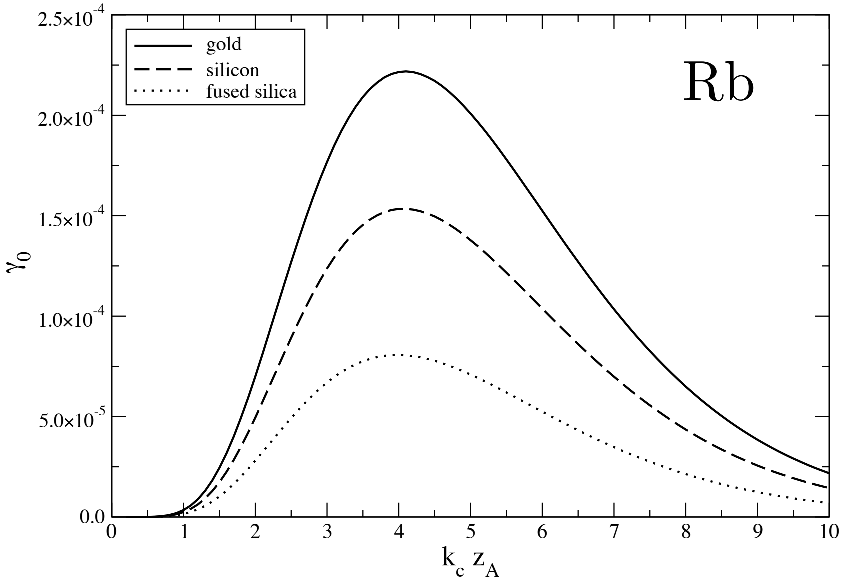


- For a single atom above a corrugated surface:
  - Sinusoidal corrugation: $\gamma_0 = -\dfrac{k_c^2\, a\, g(k_c, z_A)}{2m\,\omega_x^2}$
  - Grooved corrugation: $\boxed{\gamma_0 = -\dfrac{3k_c\, a}{2\pi m\,\omega_x^2\, s} \sum_{n=1}^{\infty} (-1)^{n+1} (1 - \cos(nk_c s/2))\, g(nk_c, z_A)}$

- Assuming PFA, the frequency shift γ_0 should vanish since the potential is locally flat on top of the plateau. Indeed, it is very small for $k_c z_A < 1$

- As increases $k_c z_A$, γ_0 develops a peak and then exponentially decreases as the atom-surface separation grows.

- The maximal frequency shift decreases as the atom-surface distance grows, reaching values $\gamma_0 < 10^{-5}$ for distances $z_A > 3\mu\mathrm{m}$



$z_A = 2\mu\mathrm{m}$ $\quad \omega_x/2\pi = 229\,\mathrm{Hz}$

$s = \lambda_c/2$ $\quad a = 250\mathrm{nm}$

Frequency shift for BEC

In the Thomas-Fermi approximation, $E_{\rm kin} \ll E_{\rm pot}$, the BEC density is

$$\boxed{n_0(\mathbf{r}) = g^{-1}[\mu - V_{\rm ho}(\mathbf{r})]}$$

$g = 4\pi\hbar^2 a/m$ atom-atom interactions

$\mu = (\hbar\omega_{\rm ho}/2)\,(15Na/a_{\rm ho})^{2/5}$ chemical potential

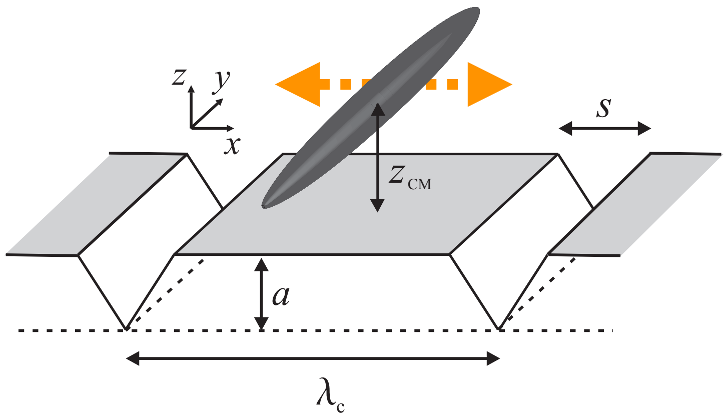
$a_{\rm ho} = (\hbar/m\omega_{\rm ho})^{1/2}$ h.o. ground state width

$\omega_{\rm ho} = (\omega_x\omega_y\omega_z)^{1/3}$ h.o. effective frequency

- Axially-symmetric cigar-shaped BEC $(\omega_y \ll \omega_x = \omega_z)$

2D density: $n_0(x,z) = \dfrac{15}{6\pi}\dfrac{1}{R^5}\left[R^2 - (x^2+z^2)\right]^{3/2}$

R is the Thomas-Fermi radius



- Relative frequency shift γ (averaging over single-atom frequency shift γ_0)

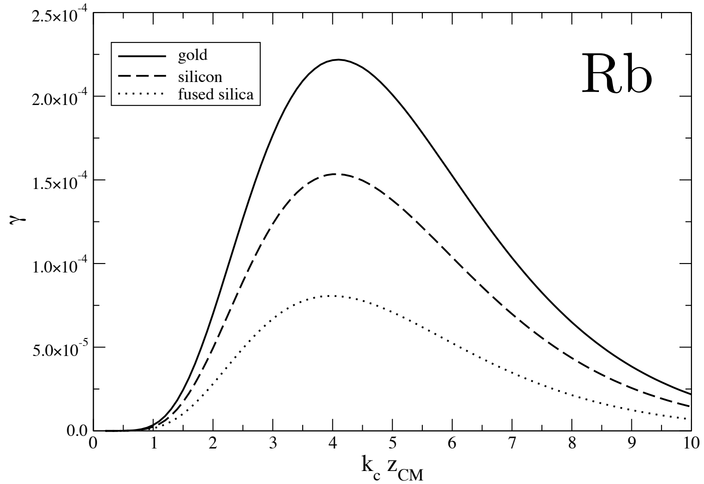
$$\gamma = -\frac{5k_c\,a}{\pi^2 m\,\omega_x^2\,s}\sum_{n=1}^{\infty}(-1)^{n+1}(1-\cos(nk_c s/2)) \times I_n(R, z_{\rm CM}, k_c)$$

$$I_n(R, z_{\rm CM}, k_c) = \frac{1}{R^5}\int_0^{2\pi} d\theta \int_0^R d\rho\,\rho\,(R^2-\rho^2)^{3/2} g(nk_c, z_{\rm CM}+\rho\sin\theta)\cos(nk_c\rho\cos\theta)$$

The single-atom case is obtained in the "point-like" limit $R \ll z_{\rm CM}, \lambda_c \Rightarrow \gamma \to \gamma_0$

Frequency shift for BEC (cont'd)

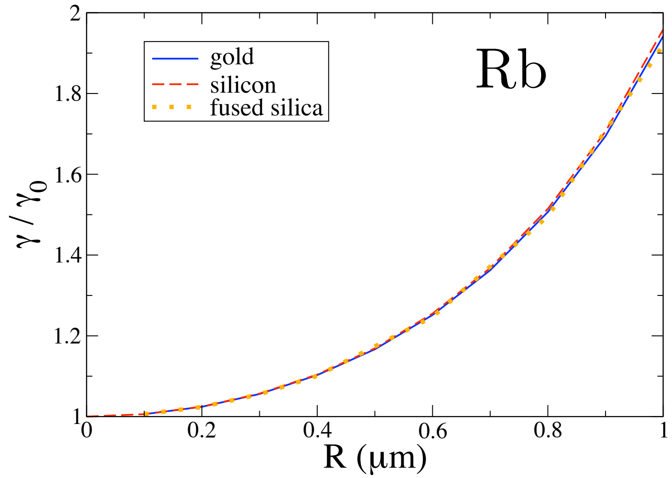
Relative frequency shift



$z_{\rm CM} = 2\mu{\rm m} \quad \omega_x/2\pi = 229\,{\rm Hz}$

$s = \lambda_c/2 \quad a = 250{\rm nm}$

Single-atom / BEC comparison



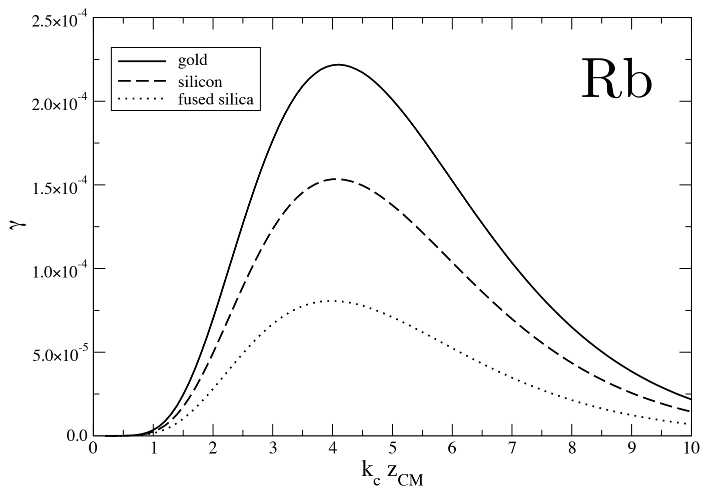
$z_{\rm CM} = 2\mu{\rm m} \quad \lambda_c = 4\mu{\rm m}$

Non-linear corrections due to finite amplitude δ_x of oscillations

$$\propto k_c^2\,\delta_x^2/8 \approx 8\% \quad (@\ \delta_x = 0.5\mu{\rm m}\ ,\ \lambda_c = 4\mu{\rm m})$$

Frequency shift for BEC (cont'd)

- Relative frequency shift



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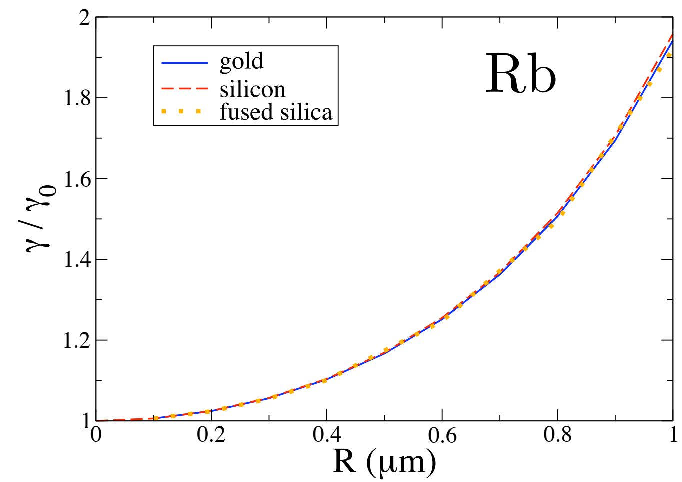

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Given the reported sensitivity $\gamma = 10^{-5} - 10^{-4}$ for relative frequency shifts from E. Cornell's experiment, we expect that beyond-PFA lateral CP forces on a BEC above a plateau of a periodically grooved silicon surface should be detectable for distances $z_{\rm CM} < 3\mu{\rm m}$, groove period $\lambda_c = 4\mu{\rm m}$, groove amplitude $a = 250{\rm nm}$, and a BEC radius of, say, $R \approx 1\mu{\rm m}$

Summary

- Novel cold atoms techniques open a promising way of investigating nontrivial geometrical effects on quantum vacuum
- Important feature of atoms: they can be used as <u>local</u> probes of quantum vacuum fluctuations
- We predict <u>large deviations from PFA</u> for the lateral Casimir-Polder force of an atom above a corrugated surface
- Non-trivial, beyond-PFA effects should be measurable using a BEC as a vacuum field sensor with available technology

For more details see:

Dalvit, Maia Neto, Lambrecht, and Reynaud, arXiv:0709.2095

Metamaterials and Casimir

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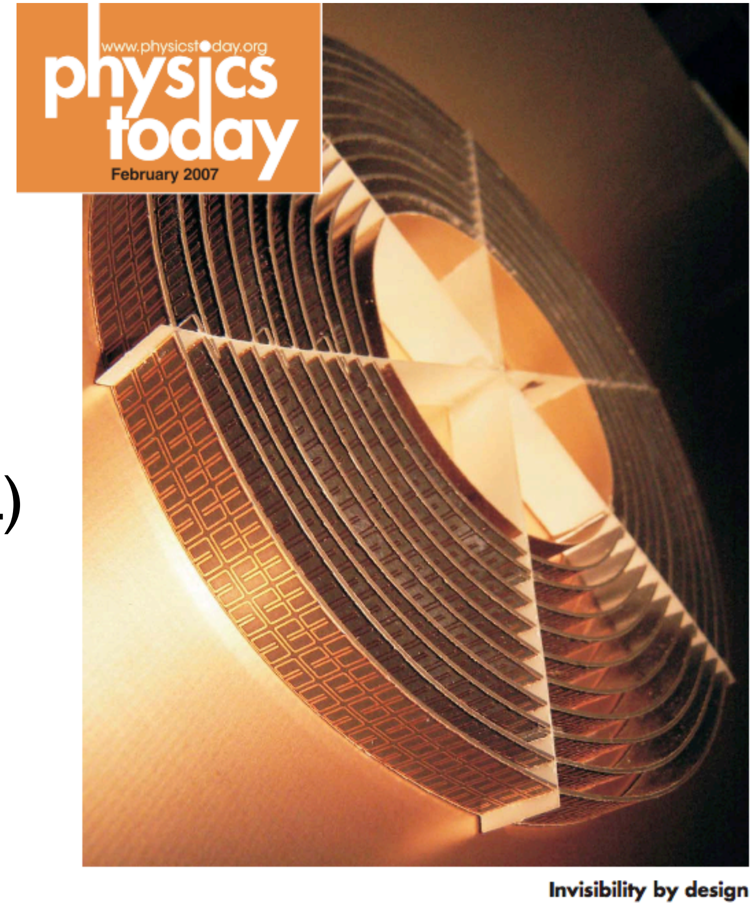
- Artificial materials for engineering the Casimir force

Ongoing work in collaboration with:

Theory: Peter Milonni (LANL)

Felipe da Rosa (LANL)

Experiment: Antoniette Taylor (CINT, LANL)



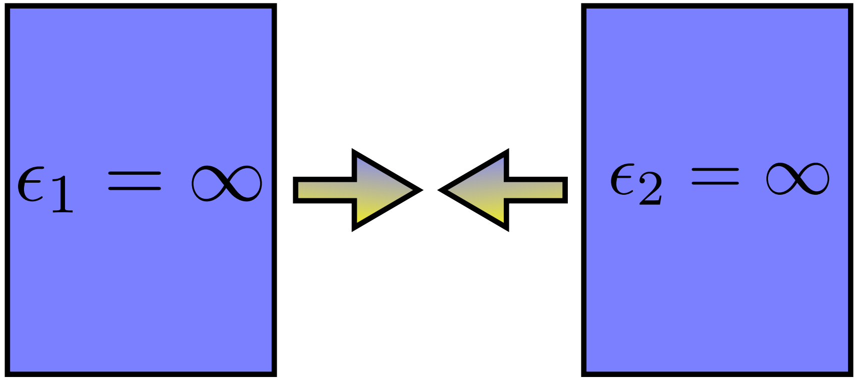
Smith et al (2007)

Casimir attraction-repulsion

- Ideal attractive limit

Casimir 1948

$$\boxed{\frac{F}{A} = +\frac{\pi^2}{240}\,\frac{\hbar c}{d^4}}$$



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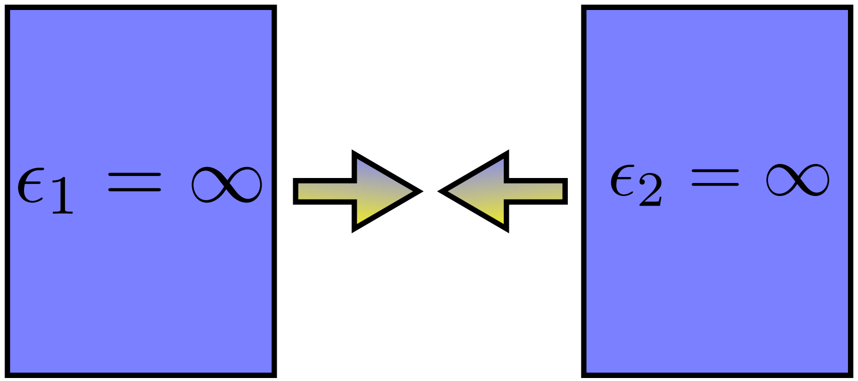
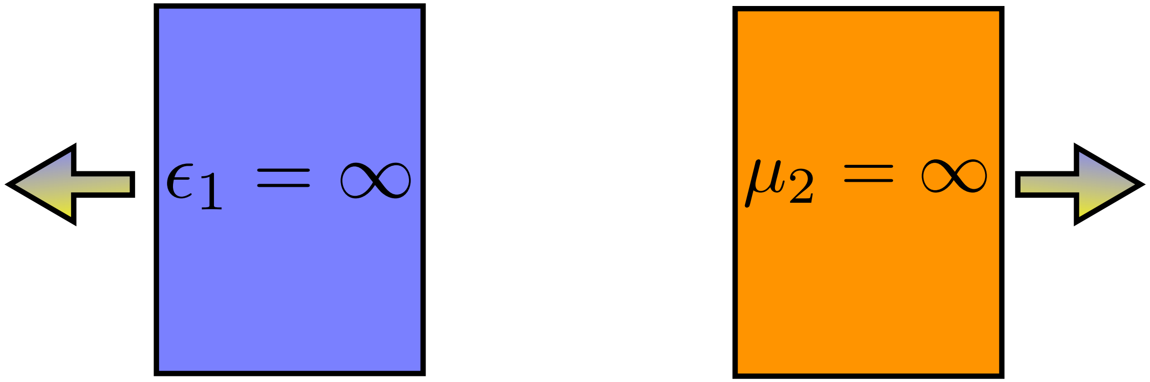


- Ideal repulsive limit

  Boyer 1974

$$\boxed{\frac{F}{A} = -\frac{7}{8}\frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$



Casimir attraction-repulsion

- ## Ideal attractive limit

Casimir 1948

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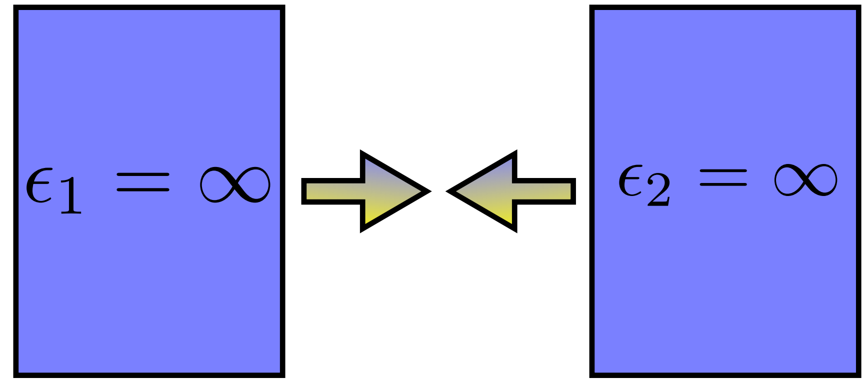

- ## Ideal repulsive limit

Boyer 1974

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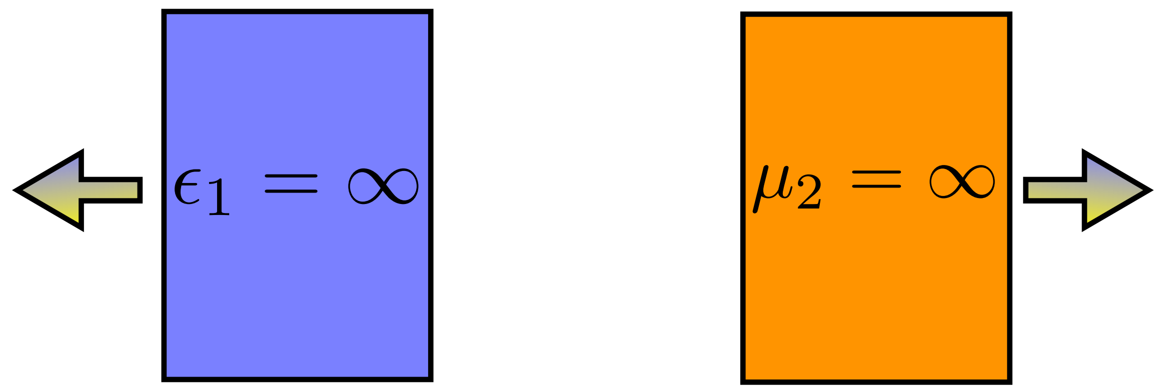

- ## Real repulsive limit

Casimir repulsion is associated with strong electric-magnetic interactions. However, natural occurring materials do NOT have strong magnetic response in the optical region, i.e. $\mu = 1$

Casimir attraction-repulsion

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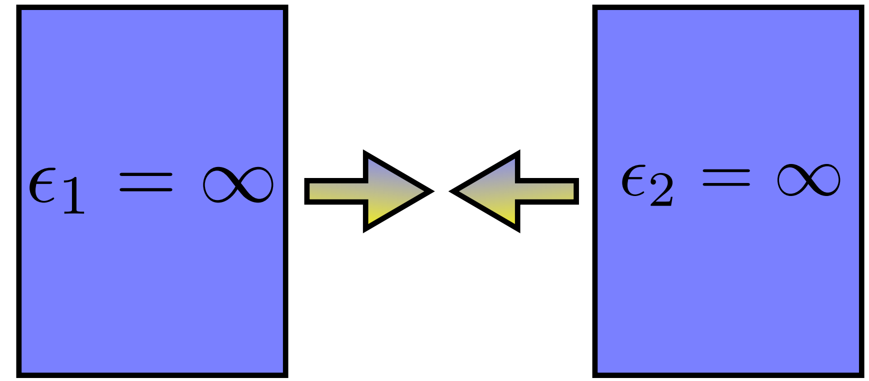
- **Ideal attractive limit**

  Casimir 1948

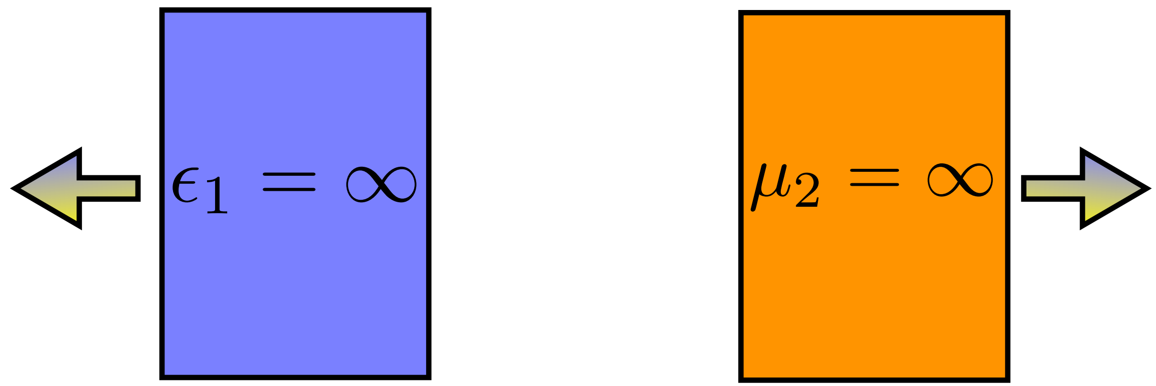
$$\boxed{\frac{F}{A} = +\frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$



- **Ideal repulsive limit**

  Boyer 1974

$$\boxed{\frac{F}{A} = -\frac{7}{8}\frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$



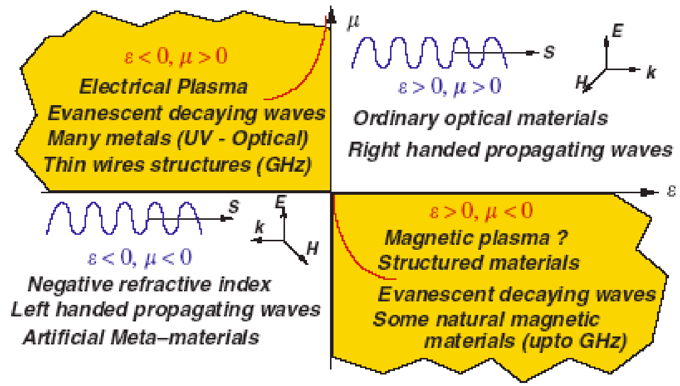
- **Real repulsive limit**

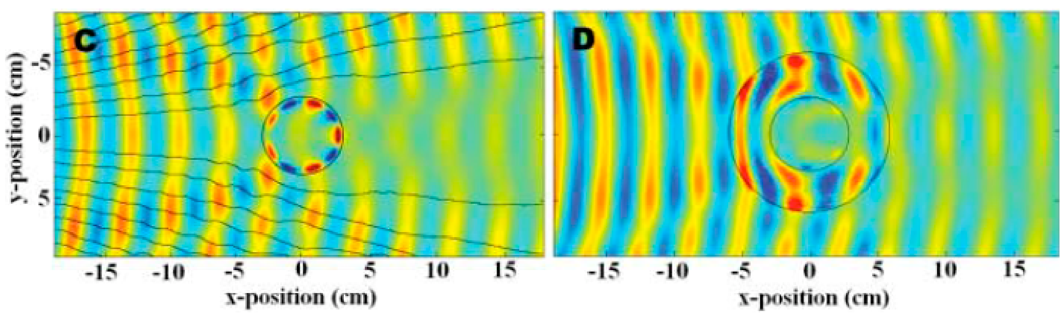
  Casimir repulsion is associated with strong electric-magnetic interactions. However, natural occurring materials do NOT have strong magnetic response in the optical region, i.e. $\mu = 1$ $\longrightarrow$ Metamaterials

Metamaterials

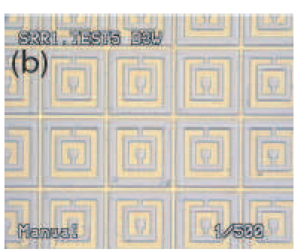
- Artificial structured composites with designer electromagnetic properties
- Macroscopic EM response described by dispersive magneto-dielectric media

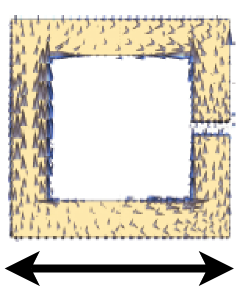
Negative refraction	Veselago (1968), Smith et al (2000)
Perfect lens	Pendry (2000)
Cloaking	Smith et al (2007)



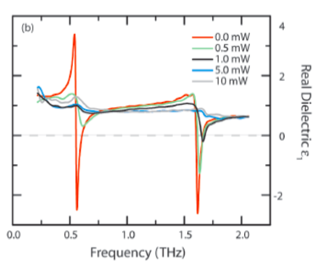


THz MMs: eg split ring resonators



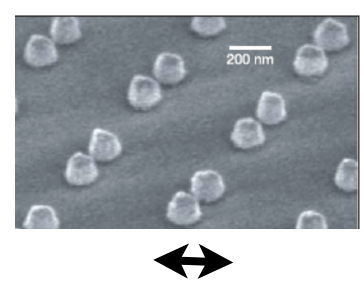


36μm

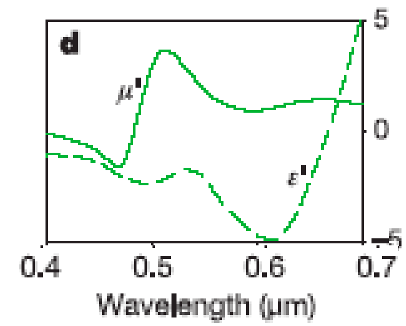


$\epsilon, \mu < 0$

Optical MMs: eg nano-pillars



200nm



Quantum levitation with MMs?

Los Alamos NATIONAL LABORATORY

Physicists have 'solved' mystery of levitation

By Roger Highfield, Science Editor
Last Updated: 1:41am BST 08/08/2007

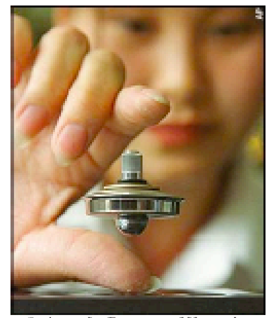
Levitation has been elevated from being pure science fiction to science fact, according to a study reported today by physicists.

In earlier work the same team of theoretical physicists showed that invisibility cloaks are feasible.

Now, in another report that sounds like it comes out of the pages of a Harry Potter book, the University of St Andrews team has created an 'incredible levitation effects' by engineering the force of nature which normally causes objects to stick together.

Professor Ulf Leonhardt and Dr Thomas Philbin, from the University of St Andrews in Scotland, have worked out a way of reversing this phenomenon, known as the Casimir force, so that it repels instead of attracts.

Their discovery could ultimately lead to frictionless micro-machines with moving parts that levitate But they say that, in principle at least, the same effect could be used to levitate bigger objects too, even a person.



In theory the discovery could be used to levitate a person

The Casimir force is a consequence of quantum mechanics, the theory that describes the world of atoms and subatomic particles that is not only the most successful theory of physics but also the most baffling.

The force is due to neither electrical charge or gravity, for example, but the fluctuations in all-pervasive energy fields in the intervening empty space between the objects and is one reason atoms stick together, also explaining a "dry glue" effect that enables a gecko to walk across a ceiling.

Now, using a special lens of a kind that has already been built, Prof Ulf Leonhardt and Dr Thomas Philbin report in the New Journal of Physics they can engineer the Casimir force to repel, rather than attact.

Because the Casimir force causes problems for nanotechnologists, who are trying to build electrical circuits and tiny mechanical devices on silicon chips, among other things, the team believes the feat could initially be used to stop tiny objects from sticking to each other.

Prof Leonhardt explained, "The Casimir force is the ultimate cause of friction in the nano-world, in particular in some microelectromechanical systems.

Such systems already play an important role - for example tiny mechanical devices which triggers a car airbag to inflate or those which power tiny 'lab on chip' devices used for drugs testing or chemical analysis.

Micro or nano machines could run smoother and with less or no friction at all if one can manipulate the force." Though it is possible to levitate objects as big as humans, scientists are a long way off developing the technology for such feats, said Dr Philbin.

The practicalities of designing the lens to do this are daunting but not impossible and levitation "could happen over quite a distance".

Prof Leonhardt leads one of four teams - three of them in Britain - to have put forward a theory in a peer-reviewed journal to achieve invisibility by making light waves flow around an object - just as a river flows undisturbed around a smooth rock.

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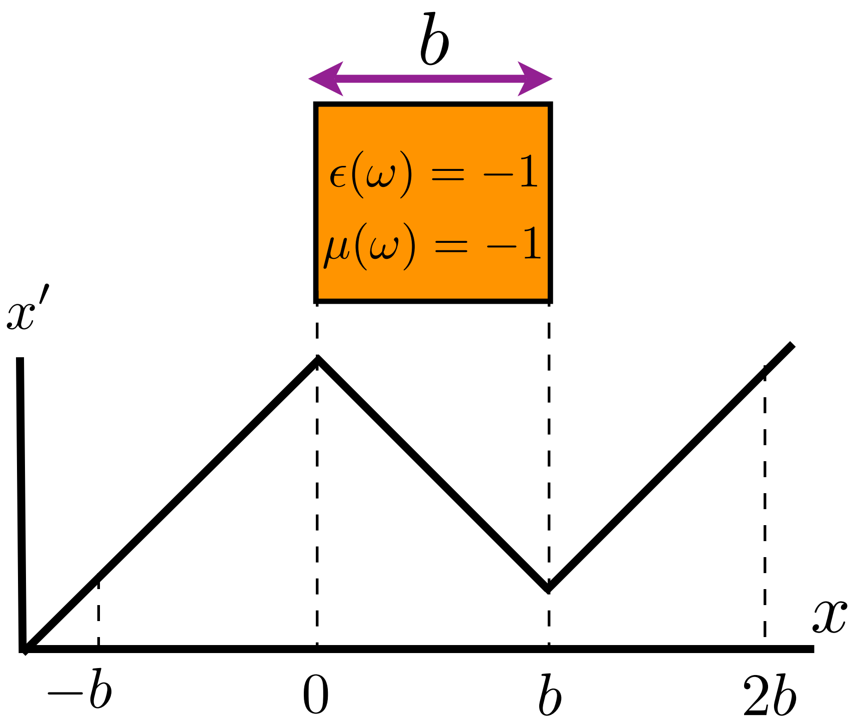
"In theory the discovery could be used to levitate a person"

Quantum levitation with MMs?

Transformation media

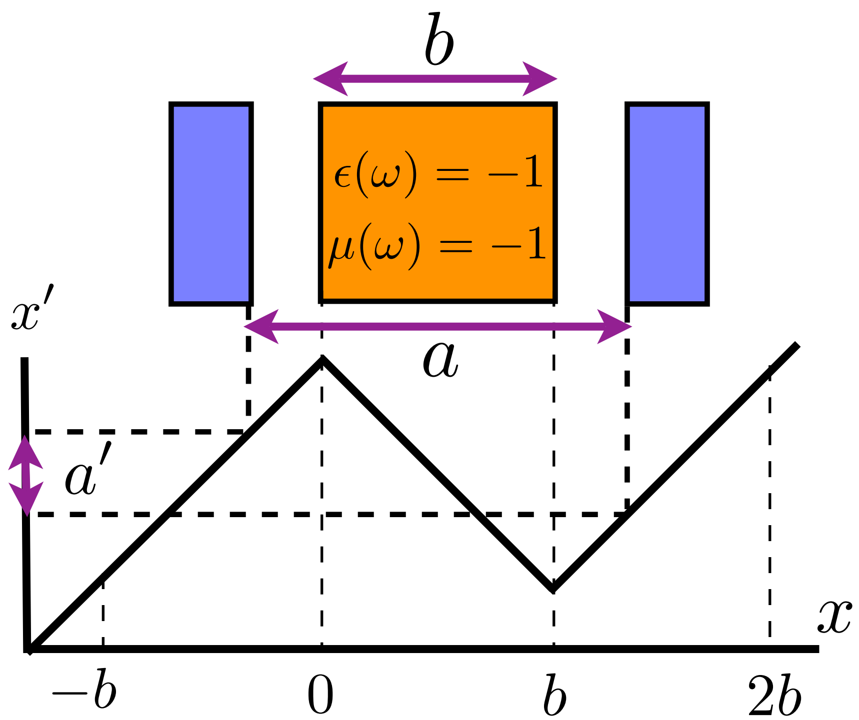
Leonhardt et al (2007)



Perfect lens: EM field in -b<x<0 is mapped into x'. There are two images, one inside the device and one in b<x<2b.

Quantum levitation with MMs?

Transformation media — Leonhardt et al (2007)



Perfect lens: EM field in -b<x<0 is mapped into x'. There are two images, one inside the device and one in b<x<2b.

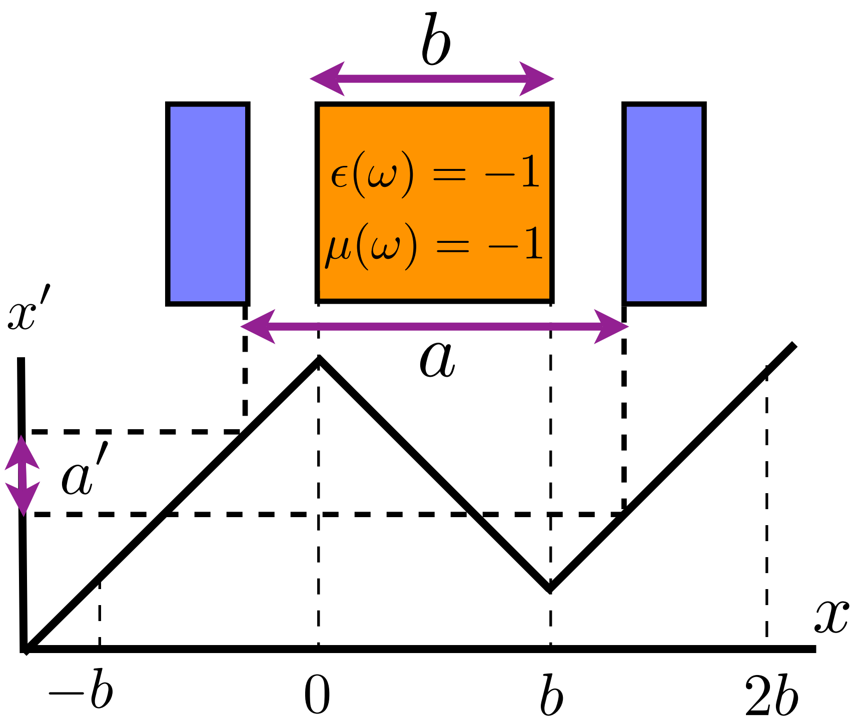
Casimir cavity: $a' = |a - 2b|$

When a< 2b (plates within the imaging range of the perfect lens)

$\Rightarrow$ $f = -\frac{\partial U}{\partial a'}\frac{\partial a'}{\partial a} = +\frac{\hbar c \pi^2}{240 a'^4}$ $\Rightarrow$ Repulsion

Quantum levitation with MMs?

Transformation media — Leonhardt et al (2007)



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$$\Rightarrow f = -\frac{\partial U}{\partial a'}\frac{\partial a'}{\partial a} = +\frac{\hbar c \pi^2}{240 a'^4} \Rightarrow$$ Repulsion

For real materials, however .....

- According to causality, no passive medium ($\epsilon"(\omega) > 0$) can sustain $\epsilon, \mu \simeq -1$ over a wide range of frequencies. In fact, $\epsilon(i\xi), \mu(i\xi) > 0$
- Leonhardt proposes to use an active MM ($\epsilon"(\omega) < 0$) in order to get repulsion. But then the whole approach breaks down, as real photons would be emitted into the quantum vacuum.

Metamaterials for Casimir

Drude-Lorentz model:

$$\epsilon_\alpha(\omega) = 1 - \frac{\Omega_{E,\alpha}^2}{\omega^2 - \omega_{E,\alpha}^2 + i\Gamma_{E,\alpha}\omega}$$

$$\mu_\alpha(\omega) = 1 - \frac{\Omega_{M,\alpha}^2}{\omega^2 - \omega_{M,\alpha}^2 + i\Gamma_{M,\alpha}\omega}$$

Typical separations

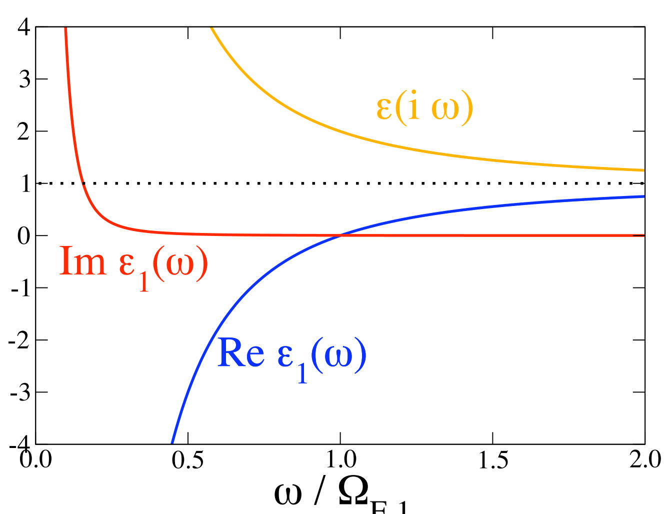
$$d = 200 - 1000 \text{ nm}$$

Infrared-optical frequencies

$$\Omega/2\pi = 5 \times 10^{14} \text{ rad s}^{-1}$$

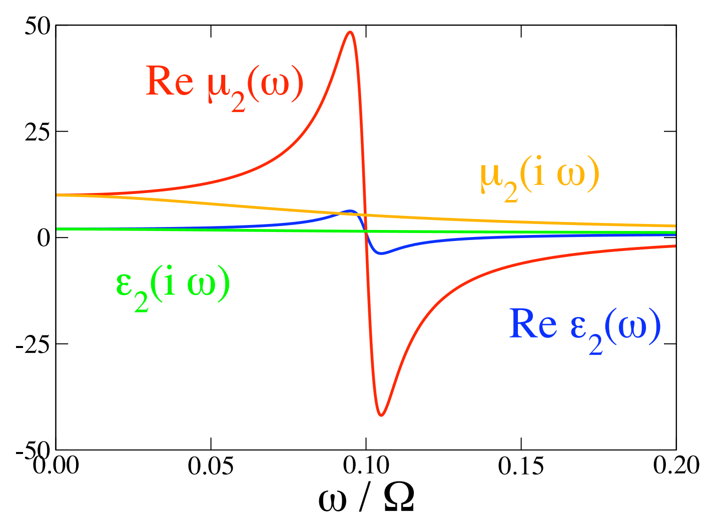
Drude metal (Au)

$\Omega_E = 9.0$ eV $\quad \Gamma_E = 35$ meV



Metamaterial

Re $\epsilon_2(\omega) < 0 \quad$ Re $\mu_2(\omega) < 0$



$$\Omega_{E,2}/\Omega = 0.1 \quad \Omega_{M,2}/\Omega = 0.3$$

$$\omega_{E,2}/\Omega = \omega_{M,2}/\Omega = 0.1$$

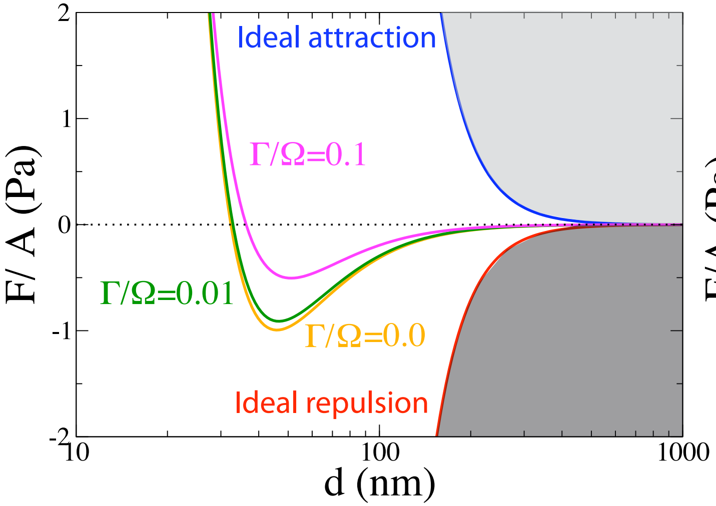
$$\Gamma_{E,2}/\Omega = \Gamma_{M,2}/\Omega = 0.01$$

Metamaterials for Casimir

Los Alamos NATIONAL LABORATORY

Drude metal (Au)

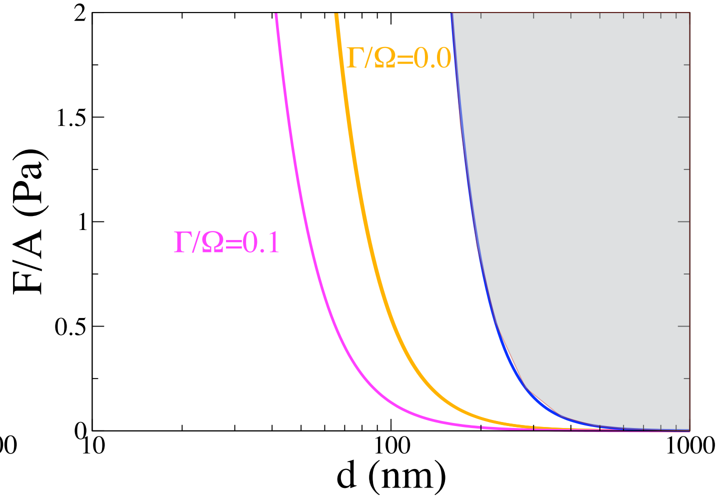
Metamaterial



Repulsion-attraction

Metamaterial

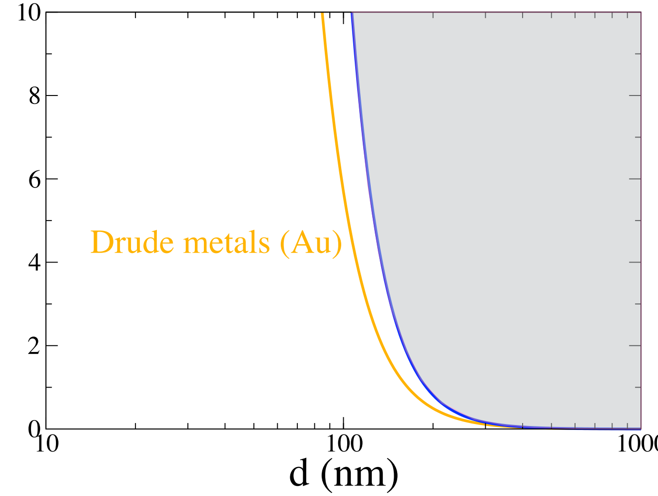
Metamaterial



Only attraction

Drude metal (Au)

Drude metal (Au)



Only attraction

A slab made of Au ($\rho = 19.3\ \mathrm{gr/cm^3}$) of width $\delta = 1\mu\mathrm{m}$ could levitate in front of one of these MMs at a distance of $d \approx 110\ \mathrm{nm}$!!!

Casimir and metamaterials, Henkel et al (2005)

Casimir and surface plasmons, Intravaia et al (2005)

van der Waals in magneto-dielectrics, Spagnolo et al (2007)

Summary

- Metamaterials can strongly influence the quantum vacuum, providing a route towards quantum levitation.

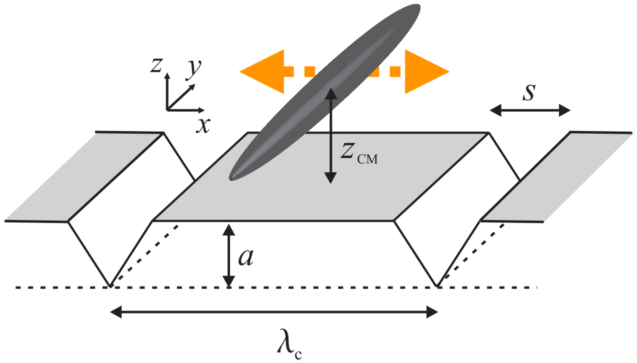
- Build MMs with strong magnetic response at infrared-optical frequencies, corresponding to gaps between 200 nm and 10 microns.

- Ongoing theory-experimental work at LANL to realize strongly modified / repulsive Casimir forces with metamaterials.

General conclusions

Casimir forces: still surprising after 60 years

☑ Non-trivial geometry effects



☑ Non-trivial materials effects